# MISCELLANEOUS.

## SOLUTIONS OF PROBLEMS.

**28.** Proposed by "IAGO"—(The late DR. JAMES MATTESON, DeKalb Center, Illinois.)

If 9 gentlemen, or 15 ladies, will eat 17 apples in 5 hours, and 15 gentlemen and 15 ladies can eat 47 apples of a similar size in 12 hours, the apples growing uniformly; how many boys will eat up 360 apples in 60 hours, admitting that 120 boys can eat the same number as 18 gentlemen and 26 ladies ?  *F. P. Matz.*

Solution by Professor P. S. BERG, Larimore, North Dakota.

1.  Call the original size of an apple an "apple unit."
2.  Call the growth of 1 apple in 1 hour one "unit of growth."
3.  $32\frac{1}{7}$ boys will in the same time eat as many as 9 gentlemen or 15 ladies.
4.  $\therefore$ $32\frac{1}{7}$ boys in 5 hours or $160\frac{5}{7}$ boys in 1 hour$=17$ "apple units"$+85$ "units of growth."
5.  $\therefore$ $85\frac{5}{7}$ boys in 12 hours or $1028\frac{4}{7}$ boys in 1 hour$=47$ "apple units"$+564$ "units of growth."
6.  $\therefore$ $(4)\times2\frac{1}{1}\frac{3}{7}=444\frac{3}{1}\frac{9}{7}$ boys in 1 hour$=47$ "apple units"$+235$ "units of growth."
7.  $\therefore$ $(4)\times6\frac{5}{3}\frac{4}{5}=1066\frac{4}{1}\frac{6}{5}$ boys in 1 hour$=112\frac{5}{8}$ "apple units"$+564$ "units of growth."
8.  $\therefore$ $(5)-(6)=584\frac{2}{1}\frac{9}{5}$ boys in 1 hour$=329$ "units of growth."
9.  $\therefore$ $(7)-(5)=37\frac{9}{1}\frac{1}{5}$ boys in 1 hour$=65\frac{5}{8}$ "apple units."
10.  $\therefore$ $(9)\times15\frac{9}{2}\frac{9}{0}=584\frac{2}{1}\frac{9}{5}$ boys in 1 hour$=1016\frac{6}{1}\frac{1}{0}\frac{}{0}$ "apple units."
11.  $\therefore$ $329$ "units growth"$=1016\frac{6}{1}\frac{1}{0}\frac{}{0}$ "apple units,
        1 "unit growth"$=3\frac{8}{1}\frac{}{0}\frac{}{0}$ "apple units."
12.  360 apples in 60 hours$=360$ "apple units"$+21600$ "units growth."
13.  360 "apple units"$=116\frac{5}{3}\frac{6}{0}\frac{6}{9}$ "units growth."
14.  $\therefore$ 360 apples in 60 hours$=116\frac{5}{3}\frac{6}{0}\frac{6}{9}+21600=21716\frac{5}{3}\frac{6}{0}\frac{6}{9}$ "units growth."
15.  From (8) 1 boy in 1 hour$=\frac{119\times329}{69525}$ "units growth."
16.  $\therefore$ 1 boy in 60 hours$=\frac{1}{4}\frac{5}{3}\frac{6}{3}\frac{6}{5}\frac{0}{}\frac{4}{}$ "units growth."
17.  $\therefore$ $21716\frac{5}{3}\frac{6}{0}\frac{6}{9}\div\frac{1}{4}\frac{5}{3}\frac{6}{3}\frac{6}{5}\frac{0}{}\frac{4}{}=643$ boys.

**29.** Proposed by ALEXANDER MACFARLANE, M. A., D. Sc., LL. D., Professor of Electrical Engineering in Lehigh University, South Bethlehem, Pennsylvania.

A rectangular room has the four walls, the floor, and the ceiling covered with mirrors; a candle is placed inside the room: find a formula which will express all the images.

Solution by G. B. M. ZERR, A. M., Ph. D., Professor of Mathematics and Applied Science, Texarkana College, Texarkana, Arkansas-Texas.

Regard the candle as a luminous point.   Then, since there are three sets of parallel mirrors, we have, from elementary optics, three sets of an infinite

number of images situated on three straight lines at right angles to one another, and intersecting at the bright point.

These mirrors are also inclined to one another at right angles. Let $A^\circ = \dfrac{\pi}{2} =$ the angle of inclination of the mirrors, $a^\circ$, $b^\circ$ the angles made by the candle with two of the mirrors.

Then $\dfrac{360^\circ - (a^\circ + b^\circ)}{A^\circ} = \dfrac{360^\circ - 90^\circ}{90^\circ} = 3 =$ the number of images due to two of the mirrors inclined at $90^\circ$. There are twelve such sets of inclined mirrors, but of the 36 images formed, 18 are repeated. $\therefore$ $\frac{1}{2}$ of 12 of $3 = 18$ images due to the inclined mirrors.

$\therefore \dfrac{12 \left\{ 2\pi - (a^\circ + b^\circ) \right\}}{\pi}$, is the formula for the images due to the inclined mirrors, where $a^\circ + b^\circ = \dfrac{\pi}{2}$.

30. Proposed by R. J. ADCOCK, Larchland, Warren County, Illinois.

When the sum of the distances of a point of a plane surface, from all the other points, is a minimum, that point is the center of gravity of the plane surface.

I. Proof by G. B. M. ZERR, A. M., Ph. D., Professor of Mathematics and Applied Science, Texarkana College, Texarkana, Arkansas-Texas.

Let $(x_1, y_1)$ be the point, $(x, y)$ any other point, $S$ the sum of the distances of $(x, y)$ from $(x_1, y_1)$.

Then $S = \int_{x_1}^{x_2} \int_{y_1}^{y_2} \sqrt{(x - x_1)^2 + (y - y_1)^2}\, dx\, dy$.

Let $\int_{x_1}^{x_2} \int_{y_1}^{y_2}$ be represented by $\int$, and $dx\, dy$ by $dA$.

$\therefore S = \int \sqrt{(x - x_1)^2 + (y - y_1)^2}\, dA = \int D\, dA$.

For a minimum, $\dfrac{dS}{dx_1} = \dfrac{(x - x_1)dA}{D} = 0$, $\dfrac{dS}{dy_1} = \dfrac{(y - y_1)dA}{D} = 0$.

$\therefore (x - x_1)dA = 0, (y - y_1)dA = 0.$ $\therefore x_1 = \dfrac{\int x\, dA}{\int dA}$, $y_1 = \dfrac{\int y\, dA}{\int dA}$.

$\therefore x_1 = \dfrac{\int \int x\, dx\, dy}{\int \int dx\, dy}$, $y_1 = \dfrac{\int \int y\, dx\, dy}{\int \int dx\, dy}$.